Signal Analysis & Communication ECE 355

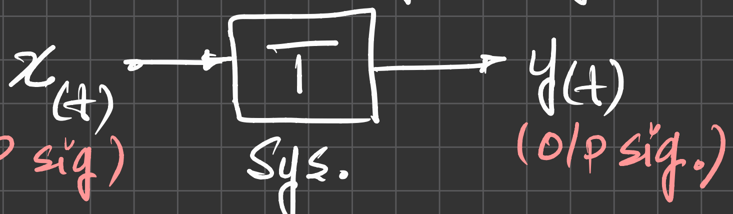
Ch 1-2: Properties of Signals

Lecture 2

11-09-2023

Recap:

① Introduction to Sig & Sys.



T is the transformation applied to $x_{(t)}$ which produces $y_{(t)}$

② Introduction to CT & DT Sig. ($x_{(t)}$ / $x_{[n]}$)

③ Size of Sig. (Energy / Average Power)

Continuing with ③:

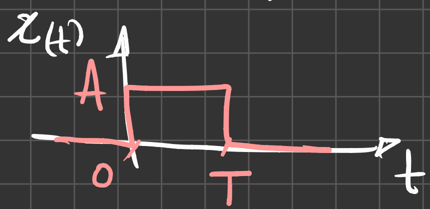
- With the mathematical def^{n}s of Energy & Power, we can identify three important classes of signals.

I. Signals with finite total energy ($E_\infty < \infty$)

- Such a sig. must have zero avg. power.

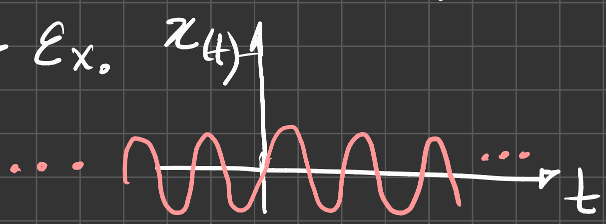
$$P_\infty = \lim_{T \to \infty} \frac{E_\infty}{2T} = 0 \quad - ①$$

- Ex.



II. Signals with finite average Power ($P_\infty < \infty$)

- If $P_\infty > 0$ in eq. ① $\Rightarrow E_\infty = \infty$
- Ex.



III. Signals for which neither P_∞ or E_∞ are finite

- Ex. $x_{(t)} = t$

ch. 1·2·1 SIGNAL TRANSFORMATION
(with respect to independent variable)

- Sig. transformation (by a sys.) is a central concept in this course
- Here, we focus sig. transformations w.r.t. the independent variable
- Apart from various applications, these transformations allow us to introduce several basic properties of signals & systems, which play an important role in characterizing important classes of systems.

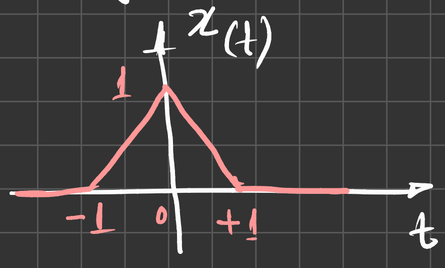
① Time-shift

- we can time-shift a CT/DT sig. $x(t)\,|\,x[n]$ by $t_0 \in \mathbb{R}\,/\,n_0 \in \mathbb{Z}$ to obtain $x(t-t_0)\,|\,x[n-n_0]$

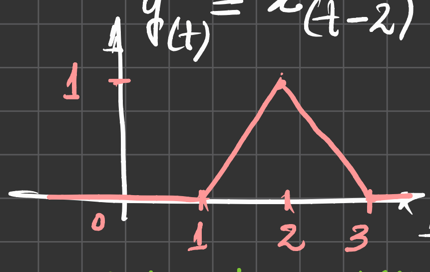
$$x(t) \longrightarrow y(t) = x(t-t_0) \qquad t_0 \in \mathbb{R}$$
$$x[n] \longrightarrow y[n] = x[n-n_0] \qquad n_0 \in \mathbb{Z}$$

- $y(t)\,|\,y[n]$ is identical in shape but shifted relative to $x(t)\,|\,x[n]$

a) Delayed: $t_0\,|\,n_0 \rightarrow +ve$



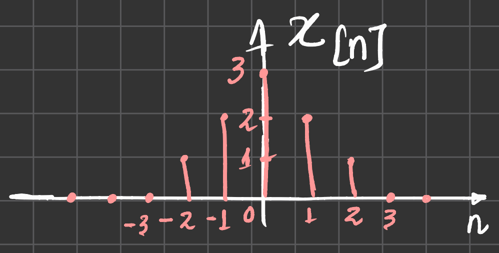
$\longrightarrow$



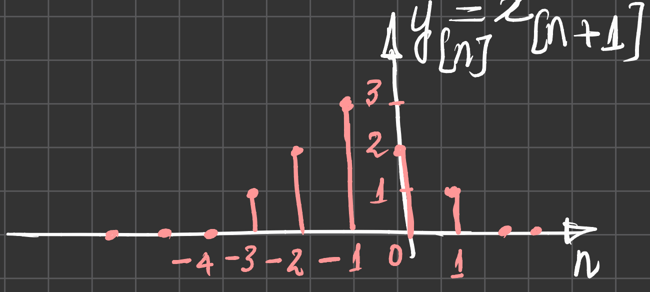
Delayed → shifted right

(check: $y(2) = x(2-2) = x(0)$)

b) Advanced: $t_0\,|\,n_0 \rightarrow -ve$



$\longrightarrow$



Advanced ⟵ shifted left

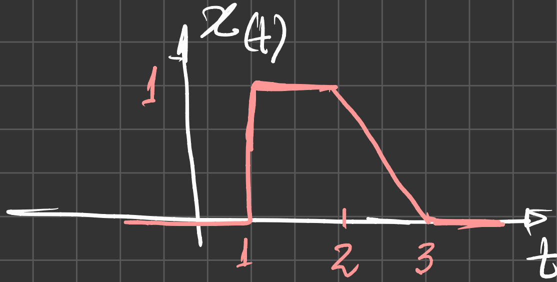
Application: Radar, Sonar, Transmission Delay

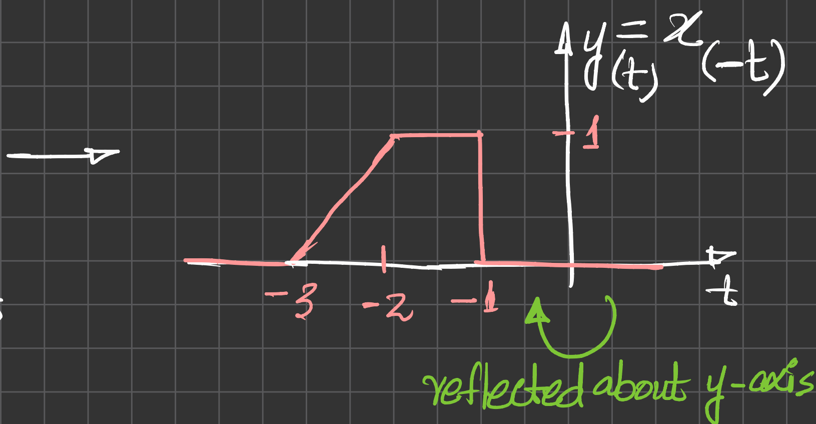
② Time Reversal:

- we can time-reverse a sig. $x(t) \mid x[n]$ by reflection about $t=0 \mid n=0$

$$x(t) \longrightarrow y(t) = x(-t)$$

$$x[n] \longrightarrow y[n] = x[-n]$$





- Similarly, for DT sig.

Application: $x(t)$ tape recording

$x(-t)$ same tape recording played backwards.

③ Time Scaling:

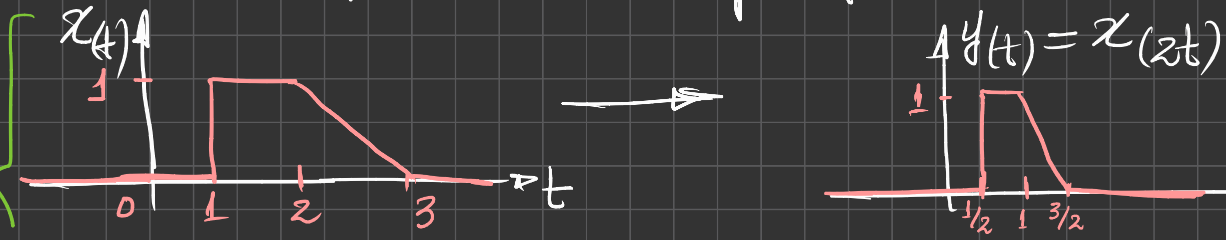
- we can time-scale a sig. $x(t) \mid x[n]$ by $\alpha \in \mathbb{R} \mid \alpha \in \mathbb{Z}$ to obtain $x(\alpha t) \mid x[\alpha n]$.

$$x(t) \longrightarrow y(t) = x(\alpha t) \qquad \alpha \in \mathbb{R}$$

$$x[n] \longrightarrow y[n] = x[\alpha n] \qquad \alpha n \in \mathbb{Z}$$

- $|\alpha| > 1$ compresses the time axis, $|\alpha| < 1$ expands time.
- if $\alpha < 0$, time effectively flips. (runs backwards)

$\alpha > 1$

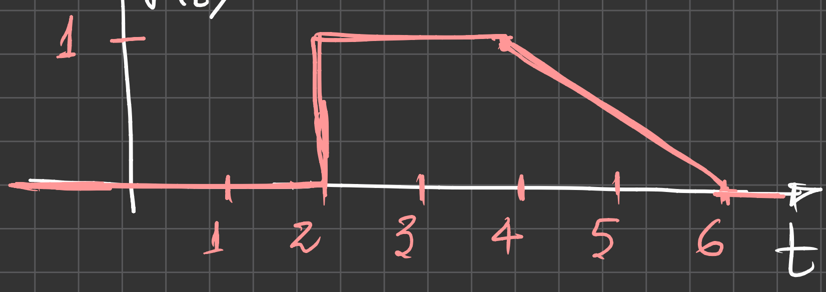


Compresses

$$x_{(t)} = \begin{cases} 1 & 1 \le t \le 2 \\ -t+3 & 2 < t \le 3 \end{cases} \longrightarrow y_{(t)} = \begin{cases} 1 & \tfrac{1}{2} \le t \le 1 \\ -2t+3 & 1 < t \le 3/2 \end{cases}$$

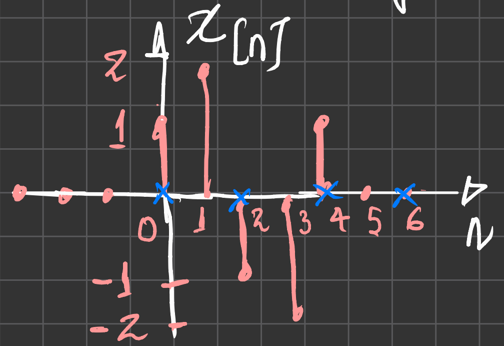
$\alpha < 1$ expands $\longrightarrow$

$y(t) = x(t/2)$

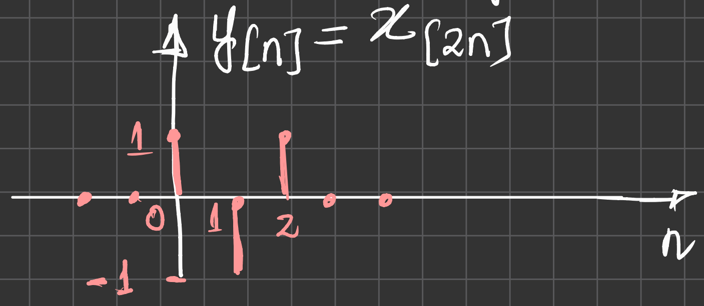


* For DT time-scaling, $y_{[n]} = x_{[\alpha n]}$; defined only if $\alpha n \in \mathbb{Z}$

$x[n]$

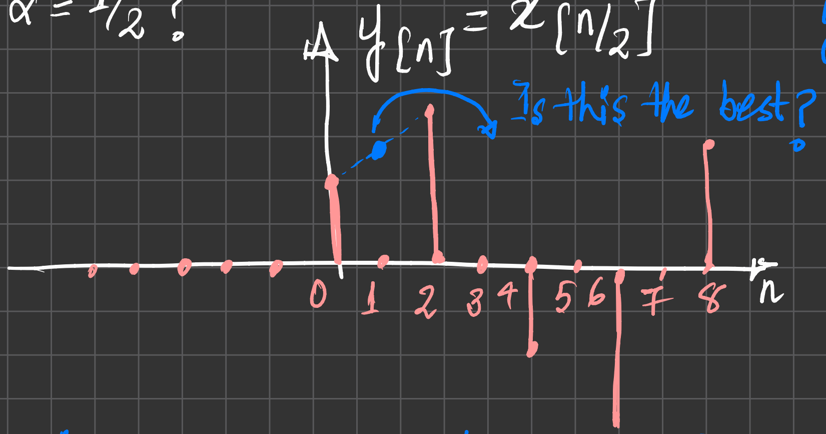


$y[n] = x[2n]$



- this operation sub-samples the sig. $x[n]$; in the above example with $\alpha = 2$, we keep only every other sample.
- what if $\alpha = 1/2$?

$y[n] = x[n/2]$



$y_{[1]} = x_{[1/2]}$ undefined!

$y_{[2]} = x_{[2/2]} = x_{[1]}$

* To fill in the missing values? "Re-sampling" (Ch. 7)
Upsampling (interpolation)

Application: Recording at twice/half the speed of the original sig.
'compress' 'expand'

④ Combine Shifting & Scaling:

- You can time-shift & time scale $x(\alpha t-\beta) \,|\, x[\alpha n-\beta]$ in two steps.

i) time shift $x(t) \,|\, x[n]$ to obtain $v(t) = x(t-\beta) \,|\, v[n] = x[n-\beta]$

ii) time scale $v(t) \,|\, v[n]$ to obtain $y(t) = v(\alpha t) \,|\, v[\alpha n]$

OR

i) time scale $x(t) \,|\, x[n]$ to obtain $w(t) = x(\alpha t) \,|\, w[n] = x[\alpha n]$

ii) time shift $w(t) \,|\, w[n]$ (by β/α) to obtain $y(t) = w(t-\frac{\beta}{\alpha}) \,|\, y[n] = w[n-\frac{\beta}{\alpha}]$

as the sig. is already scaled in (i), the shifting should consider that scaling.

1.2.2 PERIODIC SIGNALS – An important class of signals.

CT Periodic Sig.

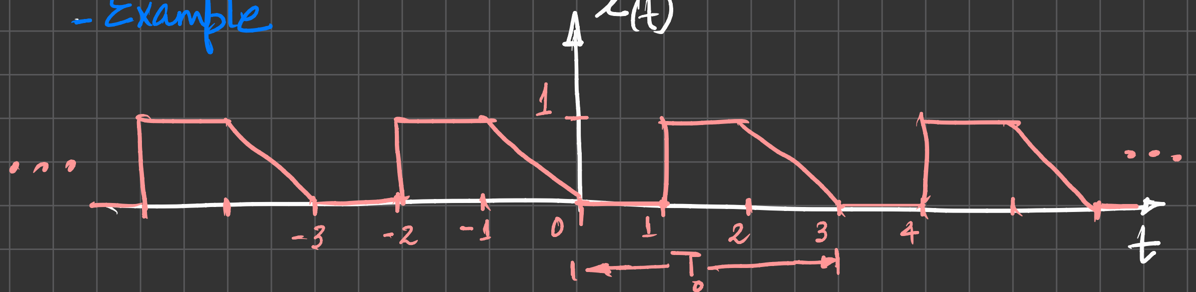
- A CT sig. $x(t)$ is periodic if

$$x(t) = x(t+T) \quad \text{for some } T > 0 \text{ and all } t \in \mathbb{R}$$

- Also, if $x(t)$ is periodic

$$x(t) = x(t+mT) \quad \text{for } m = \pm 1, \pm 2, \ldots$$

- The period is not uniquely defined, but the fundamental period is, which is the smallest positive value of T.
- If $x(t)$ is a constant sig., the fundamental period is undefined, and $x(t)$ is periodic with any choice of T.
- Example



- Examples: - ideal LC Circuit (without R - No loss) } Energy is
 - ideal mech. sys. (without friction - No loss) } Conserved

- If $x(t)$ is not periodic — Aperiodic

DT Periodic Sig.

- A DT sig. is periodic if

$$x[n] = x[n+N] \quad \text{for some integer } N \geq 1 \text{ \& for all } n \in \mathbb{Z}$$

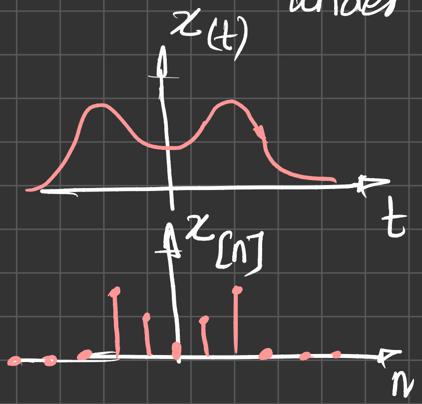
- The smallest value of N, denoted by N_0, is called the fundamental period of $x(t)$
- Any constant DT sig. is periodic for all $N \geq 1$; it therefore has fundamental period $N_0 = 1$.
- If $x[n]$ is not periodic - Aperiodic.

1.2.3 EVEN AND ODD SIGNALS - relates to sig. symmetry under time reversal.

a. Even Signals

CT: $x(-t) = x(t)$

DT: $x[-n] = x[n]$

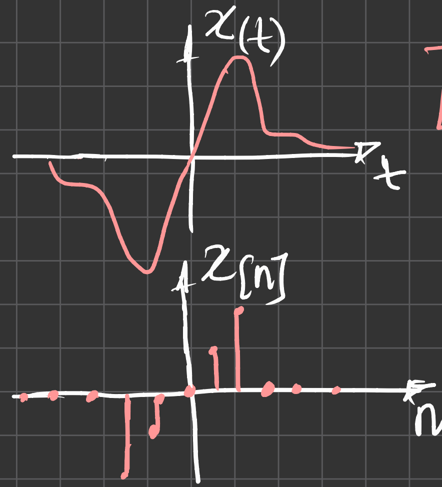


Symmetric about y-axis

b. Odd Signals

CT: $x(-t) = -x(t)$

DT: $x[-n] = -x[n]$



anti-symmetric

* Odd signals must be zero at $t=0$ since $x(0) = -x(0)$ & $x[0] = -x[0]$.

- Any sig. can be written as a sum of an even & an odd sig.

$$\mathcal{E}v\{x_{(t)}\} = x_{e(t)} = \frac{1}{2}\left[x_{(t)} + x_{(-t)}\right]$$

$$Od\{x_{(t)}\} = x_{o(t)} = \frac{1}{2}\left[x_{(t)} - x_{(-t)}\right]$$

$$\& \quad x_{e(t)} + x_{o(t)} = x_{(t)}$$

- Similarly for DT

$$x_{e[n]} = \frac{1}{2}\left[x_{[n]} + x_{[-n]}\right]$$

$$x_{o[n]} = \frac{1}{2}\left[x_{[n]} - x_{[-n]}\right]$$

$$\& \quad x_{e[n]} + x_{o[n]} = x_{[n]}$$